Chapter 12: Game Theory

Instructions: These are the notes for Chapter 12. Make sure you review the material presented here and read the additional file posted on the course webpage. **Not on Mankiw!**

Game Theory

- **Game theory** is a branch of mathematics and economics that studies the strategic interactions of agents within an environment.
- A **game** is a situation where the players' actions impact not only their outcomes, but also their rivals' outcomes.
- Games can be described by its three elements:
  1. players
  2. strategies (actions)
  3. outcomes of the actions

Examples for Games

- Apple and Samsung competing in prices during the holiday season.
- Bidding on eBay auctions.
- Sports: football, chess, tennis..
- Politics: positions taken by candidates.
- Marriage market.

Prisoner's Dilemma: A Famous Example

- Two prisoners who were partners in a crime are being questioned in separate rooms by the police.
- They have two options: confess or defect.
- Outcomes:
  - If both prisoners defect, they each get a mild 1 year sentence.

		Player 2	
		Confess	*Defect*
Player 1	*Confess*	2, 2	4, 1
	Defect	1, 4	3, 3

  - If one prisoner confesses and the other defects, the one who confesses goes free. The one who defects gets a harsh 6 year sentence.
  - If both confess, they each get a 3 year sentence.

- Payoff matrix: (first number is player 1's payoff)

Solution?

- What is the equilibrium in this game?
- A **Nash equilibrium** is a set of strategies where players have no incentive to change their actions.
  - Given player 2's choice, player 1 chooses the best action while simultaneously player 2 does the same in the nash equilibrium.
- Nash equilibrium in this example is $\{Confess, Confess\}$ which yields the payoffs 2 and 2.

To Generalize:

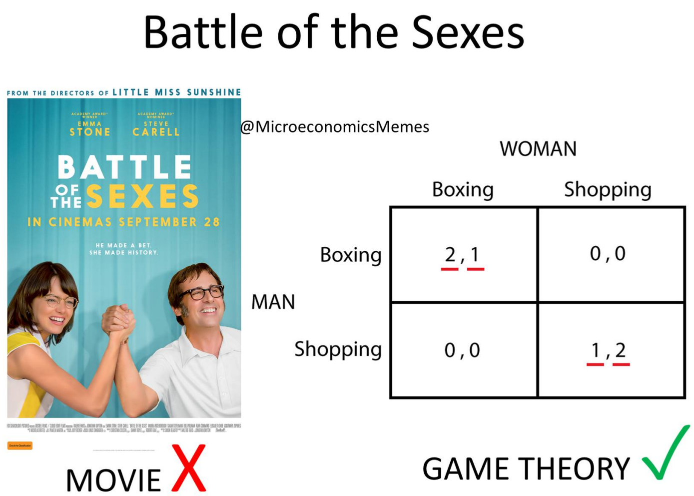
- Steps to find the Nash equilibrium
  - Step 1: Pretend you are one of the players.
  - Step 2: Assume that your "opponent" picks a particular action.
  - Step 3: Determine your best strategy (strategies), given your opponent's action Underline any best choice in the payoff matrix.
  - Step 4: Repeat Steps 2 and 3 for any other opponent strategies.
  - Step 5: Repeat Steps 1 through 4 for the other player.
  - Step 6: Any entry with all numbers underlined is the NE.

The Prisoner's Dilemma & Market Failure

- The NE in this case is leads to a worse outcome even though both players play their best strategies!
  - Classical economics says that the market outcome is the socially optimal outcome. Game theory adds: not necessarily!

- Both players would improve if they somehow coordinate and play $\{Defect, Defect\}$.

Example: Battle of the Sexes



- A couple is deciding to go out this evening.
  - The husband prefers to go to the football game.
  - The wife would rather go to the opera.
  - Both would prefer to go to the same place rather than different ones.
- Where should they go?

Setting up the payoff matrix:

		Husband: *Opera*	Husband: *Football*
Wife	*Opera*	3, 2	0, 0
	Football	0, 0	2, 3

- Two pure-strategy NE: $\{Opera, Opera\}$ and $\{Football, Football\}$.
- Both could end up being the outcome!
- Solutions to the probabilities of occurrences are found in the mixed strategy equilibrium.

Dominant Strategies

		Victoria	
		Clean	*Don't*
Albert	*Clean*	5, 5	2, 6
	Don't	6, 2	3, 3

- A **dominant strategy** is a strategy that yields the best outcome to the player no matter what strategy the other player chooses.
- *Don't Clean* is a dominant strategy for both players in the game above.
- Elimination of dominated strategies simplifies finding the solution to complicated games!

A Classroom Game: Guess the half of the average

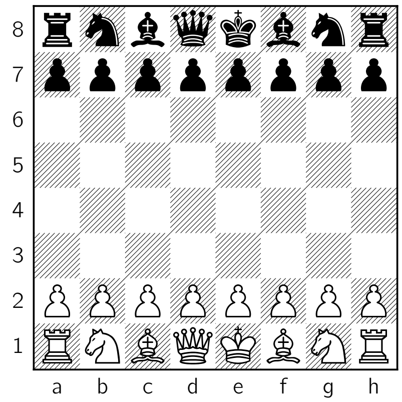
- Each player submits a number between $1 - 100$.
- The player who submits a number closest to half of the average of the numbers submitted, wins the prize!

Solution?

- Is there a dominant strategy?
  - Let's first assume everyone randomly submits numbers between 1−100. The average would be 50. The winning number would be 50/2=25.
  - However everyone knows this, and everyone submits 25. In this case, the winning number would actually be 25/2=12.5. Everyone knows this as well, the winning number would then be 12.5/2=6.25... and so on. This happens until the winning number is 1/2=0.5, so you'd submit 1 to be as close to 0.5 as possible.
- The dominant strategy for each player is to submit 1 in this game. If we assume everyone is fully-rational, this will be the outcome.

Sequential Move Games

- What if the game is played sequentially instead of simultaneously?
  - In a sequential game, players move in turns: one player makes a move and then the other players makes a move.
  - E.g. chess.



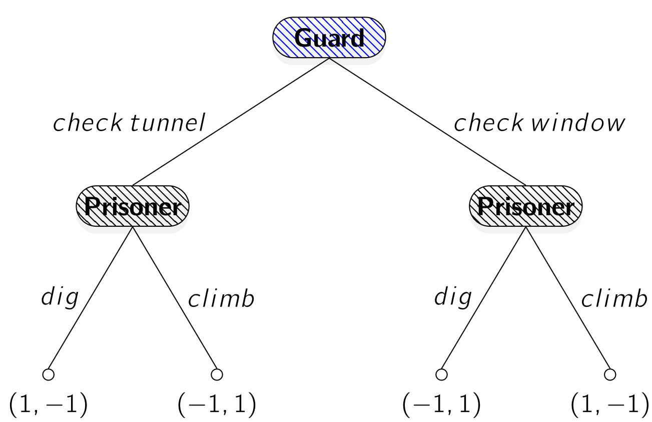
Example: A Jailbreak

- A prisoner is trying to escape the prison. His options are either to dig, or to climb over the window to escape. The guard tries to stop the prisoner by either checking for a tunnel or guarding the window.
- Assume the game is played sequentially and the guard moves first!

"Here comes the warden. Let's see you try to dance your way out of this one."

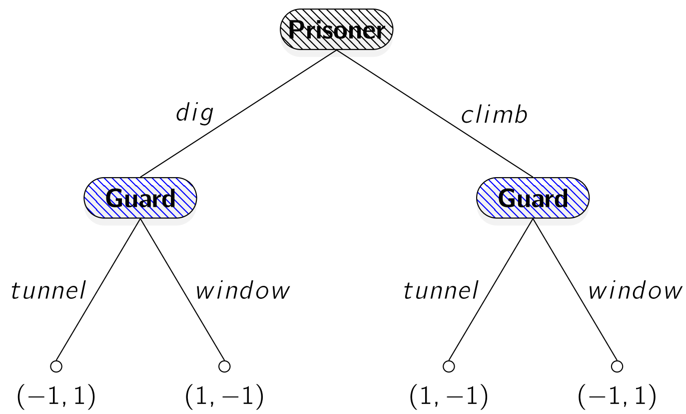
Setting up the Game Tree

- For the jailbreak game, we have (first payoff is for the guard)



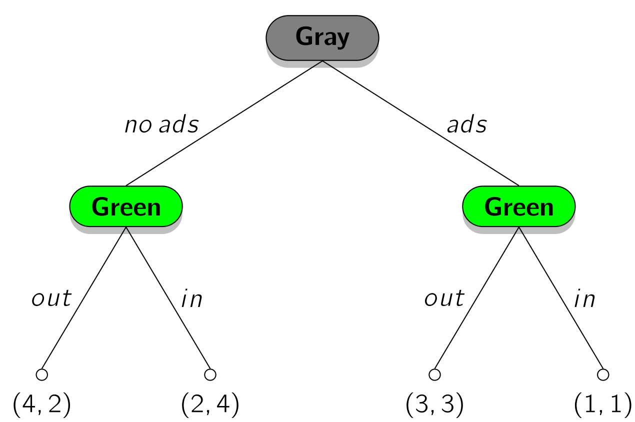
Game Theory: A Jailbreak

- The Nash equilibria of the game is
  - NE is $\{climb,\ check\,tunnel\}$ and $\{dig,\ check\,window\}$.
  - The prisoner did escape, after observing the guard's action!
- The order of sequence does matter! If the prisoner moves first, he gets caught.



Game Theory: Senate Race

- Incumbent Senator Gray will run for re-election. The challenger is Congresswoman Green.
- Senator Gray moves first, and must decide whether or not to run advertisements early on. The challenger Green moves second and must decide whether or not to enter the race.



Game Theory: Senate Race

- We can use the backward induction to find the Nash equilibrium.
  - NE is $\in \{ads,\ out\}$.
- Timing matters again! If this game was played simultaneously instead of sequentially

		Green	
		out	*in*
Gray	*no ads*	4, 2	2, 4
	ads	3, 3	1, 1

Example: Century Mark

- Two players take turns to fill out an empty jar with pennies.
- Each player puts in $1-10$ pennies (one player moves first). The player who first reaches 100 loses.
- Is there a winning strategy for any of the players?

Solution?

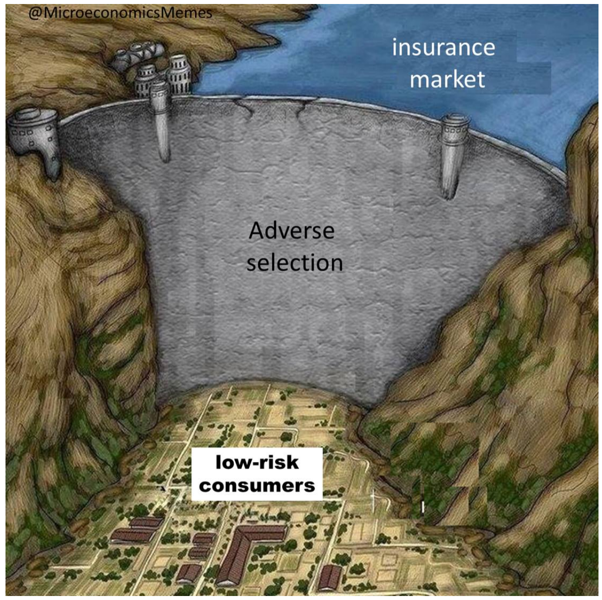
- Use backward induction to solve this sequential game.
- If you are player 1, you want to leave 99 pennies to player 2 in the last round. Because then, player 2 must put at least 1 penny hence reaches 100 and player 1 wins.
- How to make sure player 2 receives 99 pennies in the last round? Make him get 88 pennies in the round before last. If player 2 is given 88 pennies,
  - At min, he puts in 1 penny, making 89 pennies in the jar, and player 1 can put 10 pennies in the jar and wins.
  - At max, he puts in 10 pennies, making 98 pennies in the jar, and player 1 can put 1 penny and wins.
- Keep leaving 77, 66, 55...22, 11, 1 pennies to player 2.
- So in the his first turn, player 1 puts in 1 penny in the jar and forces a win, no matter what player 2 does.

2001 Nobel Prize in Economics: The Market for Lemons (Akerlof, 1970)

- Used car market: A game with imperfect information. There are 100 cars total in the market. 50 of them are lemons, other 50 are plums.
  - Plum sellers want to sell their cars for \$2,000, and lemon sellers for \$1,000.
  - Buyers would pay at most \$2,400 for a plum, \$1,200 for a lemon.
- If there was perfect information, plum cars would sell for a price \$2,400−\$2,000 and lemon cars for \$1,200−\$1,000.
- However there is no perfect information in reality. What should a buyer do in this case? Estimate the average quality of the cars in the market!
  - On average, a buyer should offer $0.5 \times 2400 + 0.5 \times 1200 =$ \$1,800.
  - However, plum owners would not sell their cars for \$1,800 (<\$2,000) and only lemon owners would agree to sell.
  - Knowing this, buyers would only offer \$1,200 at most since they know they would only be getting lemon cars.
- Only lemons remain in the market: lemons have driven out plums!

The market crashes!

- This is a problem because plum owners cannot sell their cars! If somehow we can make the plum owners can participate in the market, the society benefits overall! How? If buyers offer \$1,800 and everyone sells their cars at this price
  - Lemon owners would be happy to get 1,800−1,000=\$800 more.
  - Plum owners would be sad to get 2,000−1,800=\$200 less.
  - Overall, the society would gain \$600 of happiness from the existence of a used car market in the economy.
- Market for lemons resembles to the insurance market!
  - Lemons: high risk customers
  - Plums: low risk customers
  - If insurers average out the health expenditures, and offer this as the price of insurance, this can be too costly for the low-risk individuals, hence they would exit the market: **adverse selection.**
  - How can we fix this: mandatory health insurance!



Any other ways to fix this?

- **Signaling** is another possible solution to the adverse selection problem.
  - Recall the two sides in the used car market: buyers (less informed) and sellers (more informed).
  - The side with more information can move first and "signal" their type.
  - E.g. plum sellers can offer a warranty for their car.
    - Only plum sellers are able offer warranty and still have a positive profit.
    - If lemon sellers offer a warranty, they would have a negative profit.
    - Buyers observe whether a warranty is offered, and identify plums.

2001 Nobel Prize in Economics: Education as Signaling (Spence, 1973)

- Suppose a firm wants to hire workers.
- 2 types of workers: high ability workers and low ability workers.
  - If there was perfect information, firm would offer a wage w_H to high ability workers and a wage w_L to low ability workers ($w_L < w_H$).
- However, we don't have perfect information in real life. How much wage should the firm offer to a candidate worker?
  - Having no information, the firm would offer the average wage −which is too low for high ability workers. Only low ability workers accept this offer. Knowing this, firms only offer w_L for all candidates.
  - Same problem as in the used car market.

What to do?

- How can signaling solve this problem: education as a signal!
  - High ability workers can get education at no cost. Low ability workers have to pay c_E for education.
- Suppose a low worker gets education. Their payoff would be

$$\pi_{educ} = w_H - c_E \qquad\qquad \pi_{no\,educ} = w_L.$$

- If $w_L > w_H - c_E$ low ability workers get no education. High ability workers always get education since it has a zero cost.
- The firm is now able to distinguish! If a worker had attained education: offer w_H. If no education: offer w_L.

Game Theory: Summary

- Game theory is a tool that can be used to analyze and solve strategic situations.
- A game consists of players, strategies, and outcomes.
- The prisoner's dilemma is a classical example showing that pursuing self interest can actually lead to worse outcomes: market failure!
- Games can be simultaneous or sequential. When a game is sequential, who plays first does matter.
- There are many markets where the players are not perfectly informed. Adverse selection is a situation in which market participation is affected this information asymmetry: leading to bad outcomes!
- Signaling is one way to solve the adverse selection problem. Another solution is the forced market participation however this does not always improve the total welfare.